

Chapter 7

1. $P = mv = (15 \times 10^{-3} \text{ kg})(12 \text{ m/s}) = \underline{0.18 \text{ kg m/s}}$
2. $F = \frac{\Delta P}{\Delta t} = (1000 \text{ kg/s})(60,000 \text{ m/s}) = \underline{6 \times 10^7 \text{ N}}$
3. From the conservation of linear momentum $(3.4 \text{ kg})(10.0 \text{ m/s}) = (80 \text{ kg})v$
 $v = \underline{0.425 \text{ m/s}}$
(opposite to the direction of package)
4. momentum before = momentum after
 $(9000 \text{ kg})(20 \text{ m/s}) = (9000 + m)\text{kg} (4 \text{ m/s})$
 $m = \underline{36\,000 \text{ kg}}$
5. $(15\,000 \text{ kg})(18 \text{ m/s}) = (20\,000 \text{ kg})v$
 $v = 13.5 \text{ m/s}$
6. momentum before = momentum after
 $(140 \text{ kg})(3.0 \text{ m/s}) - (90 \text{ kg})(6.5 \text{ m/s}) = (230 \text{ kg})v$
 $v = -0.717 \text{ m/s}$
The tackler is pushed backwards.
7. Energy destroyed by friction of block and bullet is
 $\mu mgd = (0.28)(1.255)(9.8)(11) = mV^2/2 = (0.5)(1.255)V^2$
 $V = 7.77 \text{ m/s}$. Thus $(1.255)(7.77) = (0.015)v$.
 $v = \underline{650 \text{ m/s}}$.
8. $(0.022)(340) = (2.422)V$. $V = 3.09 \text{ m/s}$. $h = V^2/2g = (3.09)^2/2/9.8 = \underline{0.49 \text{ m}}$.
9. $0 = 57 \text{ mV} + m(3.8 \times 10^5 \text{ m/s})$. $V = \underline{-6.67 \times 10^3 \text{ m/s}}$, i.e. in opposite direction.
10. $(0.01)(400) = 2V + (0.01)(350)$. $V = \underline{0.25 \text{ m/s}}$.

11. Use conservation of momentum. $(700)(6.2 \times 10^3) = (350)v + (350)(v + 2.45 \times 10^3)$.

(a) $v = \underline{4.98 \times 10^3 \text{ m/s}}$. $V = \underline{7.43 \times 10^3 \text{ m/s}}$ in original direction

(b) $\Delta E = 0.5(350)(4.975 \times 10^3)^2 + 0.5(350)(7.425 \times 10^3)^2 - 0.5(700)(6.2 \times 10^3)^2 = \underline{5.25 \times 10^8 \text{ J}}$.

12. As the rocket gasses are expelled perpendicular to the original rocket direction, the component of velocity towards the sun does not change. The component perpendicular must equal $(120)(\tan 30^\circ) = 69.3 \text{ m/s}$.

$0 = (3300 - m)(69.3) - m(2200)$, $m = \underline{101 \text{ kg}}$.

13. $F = \Delta p / \Delta t = (0.06 \text{ kg})(65.0 \text{ m/s}) / (0.03 \text{ s}) = \underline{130 \text{ N}}$. No.

14. $\Delta p / \Delta t = (0.145 \text{ kg})[50 \text{ m/s} - (-35 \text{ m/s})] / (5 \times 10^{-4} \text{ s}) = \underline{2.47 \times 10^4 \text{ N}}$.

15. (a) $(90 \text{ kg})(5.0 \text{ m/s}) = \underline{450 \text{ kg} \cdot \text{m/s}}$.

(b) $F \Delta t = \Delta p = \underline{(450 \text{ kg} \cdot \text{m/s})}$.

(c) $\Delta p / \Delta t = \underline{450 \text{ N}}$.

16. $\Delta p = mv \sin 45^\circ - (-mv \sin 45^\circ) = \underline{2 mv \sin 45^\circ}$.

17. Area under curve is 9 squares = $9(50 \text{ N})(0.01 \text{ s}) = \underline{4.5 \text{ N} \cdot \text{s}}$. $mv = \text{Impulse}$.
 $v = (4.5 \text{ N} \cdot \text{s}) / (0.060 \text{ kg}) = \underline{75 \text{ m/s}}$.

18. This question can only be worked if we neglect energy loss at initial impact with the ground. Use conservation of energy $mg(h + 0.6 \text{ m}) - (170 \times 10^6 \text{ N/m}^2)(3 \times 10^{-4} \text{ m}^2)(0.6 \text{ m}) = 0$.
Hence $h = \underline{51.4 \text{ m}}$.

19. $(0.44)(4.5) = 0.22V + 0.44v$. Energy: $(0.5)(0.44)(4.5)^2 = (0.5)(0.22)V^2 + (0.5)(0.44)v^2$.
To avoid unnecessary algebra note $V = 0$, $v = 4.5 \text{ m/s}$ are a pair of solutions for these equations. We just need to find the other pair. If we eliminate v by using $v = 4.5 - 0.5V$ in second equation as $V = 0$ is a solution the quadratic is simple.
 $1.5V^2 - 9V = 0$, i.e. $V = \underline{6.00 \text{ m/s}}$, $v = \underline{1.50 \text{ m/s}}$.

20. $(0.300 \text{ kg})(2.5 \text{ m/s}) = (0.300 \text{ kg})v_1 + (0.600 \text{ kg})v_2$
 $(0.300 \text{ kg})(2.5 \text{ m/s})^2/2 = (0.300 \text{ kg})v_1^2/2 + (0.600 \text{ kg})v_2^2/2$

$v_1 = \underline{0.83 \text{ m/s backward}}$

$v_2 = \underline{1.67 \text{ m/s forward}}$

21. In general for equal masses, $v_1 + v_2 = V_1 + V_2$, $v_1^2 + v_2^2 = V_1^2 + V_2^2$. Clearly, $v_1 = V_1$, $v_2 = V_2$ are solutions, but the interesting case is $v_2 = V_1$, $v_1 = V_2$, i.e. first ball rebounds with velocity 3 m/s in same direction as second ball was originally moving. And second ball rebounds with velocity 2 m/s.

22. Consider the problem in the rest frame of the heavy particle, i.e. a frame moving with v_1 . The light particle hits the "wall" of the heavy particle with speed v_1 and bounces back with speed v_1 . So in this frame the heavy particle has speed zero, the light particle is rebounding with v_1 . Now transfer back to original frame. Heavy particle moves with speed v_1 , light particle with speed $2v_1$. We now prove mathematically what we argued physically. Best to use momentum and mass as variables. Momentum conservation gives, $p_1 = p_1' + p_2'$. Energy conservation ($E = mv^2/2 = (mv)^2/2m = p^2/2m$) gives $p_1^2/2m_1 = p_1'^2/2m_1 + p_2'^2/2m_2$. Thus $p_2' \simeq 2p_1m_2/m_1$, neglecting $p_2'^2/2m_1$ term as small. Thus $v_2' = 2v_1$. $p_2' = m_1v_1 - m^2v_2$, thus $v_1' \simeq v_1$ neglecting terms (m_2/m_1).

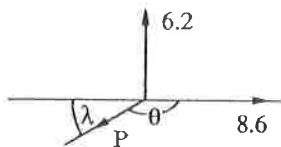
23. Proceed as in previous problem, but as now $m_1 \ll m_2$, eliminate p_2 ,
 $p_1^2/2m_1 = p_1'^2/2m_1 + (p_1 - p_1')^2/2m_2$. $p_1 \simeq -p_1'$, and hence $v_1 = -v_1'$ and $v_2' = 0$.

24. Momentum: $(0.46)v = m(v/2) + (0.46)V$.
 Energy: $(0.5)(0.46)v^2 = (0.5)m(v^2/4) + (0.5)(0.46)V^2$.

(a) Eliminating V , gives $m = \underline{1.38 \text{ kg}}$.

(b) Energy fraction is $[0.5(1.38)v^2/4]/[0.5(0.46)v^2] = \underline{0.75}$.

25.



$p = (8.6^2 + 6.2^2)^{1/2} 10^{-23} \text{ kg} \cdot \text{m/s} = \underline{10.6 \times 10^{-23} \text{ kg} \cdot \text{m/s}}$. By momentum conservation $10.6 \times 10^{-23} \cos \lambda = 8.6 \times 10^{-23}$. $\theta = \underline{144^\circ}$.

$$26. \quad (a) \quad (0.4 \text{ kg})(1.8 \text{ m/s}) = (0.4 \text{ kg})v \cos \theta + (0.4 \text{ kg})(1.1 \text{ m/s})(\cos 30^\circ) \\ (0.4 \text{ kg})(1.1 \text{ m/s})(\sin 30^\circ) = (0.4 \text{ kg})v \sin \theta.$$

$$(b) \quad v = \underline{1.01 \text{ m/s}}, \quad \theta = \underline{33^\circ}.$$

$$27. \quad \text{Applying momentum and energy conservation gives } mv = 2mV \cos \theta, \quad 2mV \sin \theta = mu, \\ mv^2/2 = 2mV^2/2 + mu^2/2. \quad \text{Solving}$$

$$(a) \quad \theta = \underline{-30^\circ}.$$

$$(b) \quad u = \underline{v/(3)^{1/2}}, \quad V = \underline{v/(3)^{1/2}}.$$

$$(c) \quad \text{Fraction of energy} = 2mV^2/mv^2 = \underline{2/3}.$$

$$28. \quad \text{Momentum must be conserved in the } x \text{ and } y \text{ directions.}$$

$$\underline{x} \quad (m)(4.0 \text{ m/s}) = mv_1'$$

$$v_1' = \underline{4.0 \text{ m/s (in the positive } x \text{ direction)}}$$

$$\underline{y} \quad m(2.5 \text{ m/s}) = m(v_2')$$

$$v_2' = \underline{2.5 \text{ m/s (in the positive } y \text{ direction)}}$$

$$29. \quad \text{Conservation of momentum perpendicular to the original direction gives} \\ (0.4 \text{ kg})V_A \sin 30^\circ = (0.6 \text{ kg})V_B \sin 53^\circ. \quad V_B/V_A = \underline{0.417}.$$

30. Since $4m_1 = m_2$, Eqs 7-a b & c become

$$v_1^2 = v_n'^2 + 4v_{He}'^2$$

$$v_1 = v_n' \cos \theta_1' + 4 v_{He}' \cos \theta_2'$$

$$0 = v_n' \sin \theta_1' + 4v_{He}' \sin \theta_2'$$

where $v_1 = 4.5 \times 10^5$ m/s and $\theta_2' = 45^\circ$.

Second and third equations become

$$v_n'^2 \cos^2 \theta_1' = v_1^2 - 8v_1 v_{He}' \cos \theta_2' + 16 v_{He}'^2 \cos^2 \theta_2' \text{ and}$$

$$v_n'^2 \sin^2 \theta_1' = 16 v_{He}'^2 \sin^2 \theta_2'. \text{ Therefore}$$

$$v_1^2 - 4v_{He}'^2 = v_n'^2 = v_1^2 + 16 v_{He}'^2 - 8v_1 v_{He}' \cos \theta_2'$$

Thus $20 v_{He}'^2 = 8v_1 v_{He}' \cos \theta_2'$, or

$$\begin{aligned} v_{He}' &= (2/5)v_1 \cos \theta_2' \\ &= (0.4)(4.5 \times 10^5) \cos 45^\circ \\ &= \underline{1.27 \times 10^5 \text{ m/s.}} \end{aligned}$$

$$\begin{aligned} v_n' &= (v_1^2 - 4v_{He}'^2)^{\frac{1}{2}} = [4.5^2 - 4(1.27)^2]^{\frac{1}{2}} \times 10^5 \\ &= \underline{3.71 \times 10^5 \text{ m/s.}} \end{aligned}$$

$$\begin{aligned} \sin \theta_1' &= -(4v_{He}'/v_n') \sin \theta_2' = -0.97 \\ \theta_1' &= \underline{-76^\circ}. \end{aligned}$$

31. Momentum: $mv = (M + m)V$.

Energy: $(M + m)V^2/2 = (M + m)gh$, where h = height swung.

Geometry: $h = l(1 - \cos \theta)$, $x = l \sin \theta$, where θ = angle deflected, i.e. $h = l - \sqrt{l^2 - x^2}$.

$$h = V^2/2g = [mv/(M + m)]^2/2/g = [(0.02)(250)/3.82]^2/2/9.8 = 0.087 \text{ m}$$

$$\text{horizontal displacement } x = (2hl - h^2)^{\frac{1}{2}} = \underline{0.63 \text{ m}}$$

32. (a) $KE = mv_1^2/2$

$$\Delta KE = KE - (1/2)(m + M)v'^2 = KE - [m^2/(m + M)]v_1'^2/2$$

$$\Delta KE/KE = \underline{1 - m/(m + M)}$$

$$(b) \Delta KE/KE = 1 - 0.01/(.25) = 0.96.$$

33. Use kinetic energy $= p^2/2m$. Let m be the smallest mass. Then as momenta are equal
 $(p^2/2m) + (p^2/3m) = 4500 \text{ J}$. Hence $p^2/2m = (3/5)(4500)\text{J} = \underline{2700 \text{ J}}$, and
 $p^2/3m = (2/5)(4500)\text{J} = \underline{1800 \text{ J}}$. The smallest mass acquired the largest energy.

34. Conservation of momentum:

$$(6.8)(9.6) = (6.8 + 8.3) v \cos \theta$$

$$(8.3)(8.4) = (6.8 + 8.3) v \sin \theta$$

$$v = \underline{6.32 \text{ m/s.}}$$

$$\theta = \underline{46.9^\circ} \text{ to original direction of first eagle.}$$

35. (a) Elastic: conservation of momentum: $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$;
 conservation of energy: $m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2$,
 or, $m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$.
 $m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2' - v_2)(v_2 + v_2')$
 But first equation implies $m_1(v_1 - v_1') = m_2(v_2' - v_2)$.
 Thus $v_1 + v_1' = v_2 + v_2'$, i.e. $e = (v_1' - v_2')/(v_2 - v_1) = 1$.
 Completely inelastic collision, $v_1' = v_2'$.
 Thus $e = (v_1' - v_2')/(v_2 - v_1) = 0$

- (b) $v_2 = v_2' = 0$ since steel plate is heavy.
 $h = v_1^2/2g$, $h' = v_1'^2/2g$.

$$e = v_1'/v_1 = \sqrt{h'/h}$$

36. $0 = 4uV_1 - 228uV_2$

$$5.3 \text{ MeV} = \frac{1}{2}(4u)V_1^2$$

$$V_1^2 = 2(5.3 \text{ MeV})/(4u)$$

$$V_1 = 1.63(\text{MeV}/u)^{\frac{1}{2}}$$

$$V^2 = 4uV_1/228u = 0.0175 V_1$$

$$KE_{T_0} = \frac{1}{2}(228 u)V_2^2$$

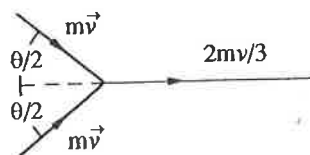
$$= 114u(0.0175 V_1)^2$$

$$= 114u(0.175)^2(1.63)^2 \text{ MeV}/u$$

$$= 0.0930 \text{ MeV}$$

$$\text{Total KE} = 5.3 \text{ MeV} + 0.093 \text{ MeV} = 5.393 \text{ MeV}$$

37. The completely inelastic collision means the two objects move off together. Hence
 $2 mv \cos \theta/2 = 2 mv/3$, i.e. $\theta = 141^\circ$.



38. (a) $(3)(6) + (2)(-4) = (3 + 2)V$. $V = 2$ m/s (in +x direction)

(b) $(3)(6) + (2)(-4) = 3v_3 + 2v_2$.

$$6 - (-4) = v_2 - v_3.$$

$$v_2 = \underline{8 \text{ m/s}}, v_3 = \underline{-2 \text{ m/s}}.$$

(c) $(3)(6) + (2)(-4) = 2 v_2$. $v_2 = \underline{5 \text{ m/s}}.$

(d) $(3)(6) + (2)(-4) = 3 v_3$. $v_3 = \underline{3.3 \text{ m/s}}.$

(e) $(3)(6) + (2)(-4) = (3)(-4) + 2 v_2$. $v_2 = \underline{11 \text{ m/s}}.$

(c) is not reasonable, because that means the two masses have to go through one another.

(d) is reasonable, the 2-kg mass is "bounced back"

(e) is not reasonable since KE after $> KE$ before.

39. $X = [(12 \text{ u})(1.13 \times 10^{-10} \text{ m}) + 16 \text{ u}(0)]/(12 + 16) = \underline{4.84 \times 10^{-11} \text{ m}}.$

40. $\bar{x} = \frac{(1200 \text{ kg} \times 3.1 \text{ m}) + (3 \times 65 \text{ kg} \times 3.85 \text{ m}) + (2 \times 65 \text{ kg} \times 2.6 \text{ m})}{(1200 + 5 \times 65) \text{ kg}}$

$$\bar{x} = \underline{3.15 \text{ m}}.$$

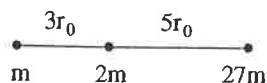
41. If your mass is (100 kg) the mass of one leg = $(100 \text{ kg})(21.5 + 9.6 + 3.4)/200 = \underline{17.25 \text{ kg}}.$

42. Position of cm from shoulder (%) is $[(6.6)(9.5) + (4.2)(25.9) + (1.7)(38.1)]/[6.6 + 4.2 + 1.7] = \underline{18.9\%}.$

43. Measuring distance from shoulder. Upper arm cm is $81.2 - 71.7 = 9.5$; lower arm cm is $81.2 - 55.3 = 25.9$; hand is $81.2 - 43.1 = 38.1$.
Assuming cm of upper arm is in middle this gives 19 as length of upper arm, 6.9 as height of lower arm cm and 19.1 as height of hand cm (above elbow, when joint is bent).
Thus $X = (170 \text{ cm}/100)[(6.6)(9.5) + (5.9)(19)]/[6.6 + 5.9] = \underline{23.8 \text{ cm}}$ from shoulder.
 $Y = (170 \text{ cm}/100)[0 + (4.2)(6.9) + (1.7)(19.1)]/[6.6 + 5.9] = \underline{8.36 \text{ cm}}.$
From elbow, (8.4 cm, 8.5 cm)

44. Measuring distances of cm from torso: they are (as in problem [43]) upper arm 9.5; lower arms 25.9; hands 38.1; upper legs $52.1 - 42.5 = 9.6$; lower legs $52.1 - 18.2 = 33.9$; and feet $52.1 - 1.8 = 50.3$.
Then $Y = [(6.6)(9.5) + (1.7)(38.1) + (21.5)(9.6) + (9.6)(33.9) + (3.4)(50.3)]/100 = \underline{9.39\%}$ below torso.
It will generally be outside the body.

45.



The spheres have mass, m , $8m$, and $27m$, assuming they are and the same density.

$$X = [0 + 3(8m) + 8(27m)]r_0/[m + 8m + 27m] = \underline{6.6r_0}$$

46. Take origin as center of raft with y north and x west. The position of CM is $x = y = (1000)(-15)/(8400) = -1.79$ m. Thus 2.53 m in SE direction.

If raft is free to move then CM does not change relative to water. If raft is fixed new $Y = (1000)[-15 + (0.5)(0.8)(4)^2]/8400 = \underline{-1.02}$ m. Thus CM is at 2.06 m 29.7° S of E.

47. $X = -(\pi R^2)(0.8R)/[\pi(2R)^2 - \pi R^2] = \underline{-0.27R}$, from the center, i.e. on the opposite direction of hole.

48. (a) Measuring from the earth,

$$R = (7.36 \times 10^{22} \text{ kg})(3.80 \times 10^8 \text{ m})/(7.36 \times 10^{22} \text{ kg} + 5.98 \times 10^{24} \text{ kg}) \\ = \underline{4.68 \times 10^6 \text{ m from earth}}$$

(b) The earth-moon system moves around the sun in an ellipse. The earth and the moon move at the (moving) earth-moon cm in ellipses.

49. (a) $X = (70)(8)/120 = \underline{4.7 \text{ m}}$ from girl.

(b) He is $8 - 2 - 4.67 = 1.33$ m from CM. Therefore $(50)L = (70)(1.33)$ where L is distance of from CM. $L = \underline{1.87 \text{ m}}$. So distance between boy and girl is 3.2 m.

(c) His original distance from CM was 3.33 m. When colliding, both he and girl are at CM, so has moved 3.3 m.

50. Final position of cm is at $2D$. Let L be distance m_H falls beyond this point.

(a) Then $m_1D = m_HL = 3m_1L$, i.e. $L = D/3$. So rocket lands $(7/3)D$ from original position.

(b) Then $m_1D = 3m_HD = m_HL$, i.e. $L = 3D$. So rocket lands $5D$ from original position.

51. When the man moves down the gondola will move up. The center of mass does not move.

$$U_{ME} = U_{MG} + U_{GE}$$

$m \rightarrow$ man, $E \rightarrow$ earth, $G \rightarrow$ gondola

Taking moment about the cm.

$$M U_{GE} = m(U - U_{GE}) \quad \text{Taking down as positive.}$$

$$M U_{GE} = m U - m U_{GE}$$

$$(M + m)U_{GE} = m U$$

$$U_{GE} = \frac{m U}{m + M} \text{ upward.}$$

If man stops, gondola stops.

52. $F = \Delta p / \Delta t = (4.3 \times 10^4)(27.8) = \underline{1.20 \times 10^6 \text{ N}}$

53. $(6000)(15) = [6000 + (4 \text{ kg/min})(80 \text{ min})]v$. $v = \underline{14.2 \text{ m/s}}$.

54. Use conservation of momentum. In x direction $mv_0 = (2/3)mv_x$. Hence $v_x = \underline{(3/2)V_0}$.
In y direction $0 = (1/3)m(2 v_0) + (2/3)mv_y$. $v_y = \underline{-v_0}$

Any two momentum vectors form a plane. Unless the third vector is in that plane it has an unbalanced component perpendicular to the plane, i.e. the resultant momentum could not be zero. But it must be from conservation of momentum as p was zero initially.

56. (a) $0 = (150 \text{ kg})(2.35 \text{ m/s}) + (2000 \text{ kg})V$
 $V = \underline{-0.176 \text{ m/s}}$ in opposite direction from astronaut.

(b) $F(0.200 \text{ s}) = (150 \text{ kg})(2.35 \text{ m/s})$
 $F = \underline{1.76 \times 10^3 \text{ N}}$

57. Momentum: $mv = MV - mv/3$. Energy: $0.5 mv^2 = 0.5 mV^2 + 0.5 mv^2/9$. Hence $M = \underline{m/2}$.

58. $(10^8 \text{ kg})(15 \text{ km/s}) = (6 \times 10^{24} \text{ kg})V.$

(a) $V = \underline{2.5 \times 10^{-16} \text{ km/s}}.$

(b) $f = (6 \times 10^{24} \text{ kg})(2.5 \times 10^{-16} \text{ km/s})^2 / (10^8 \text{ kg}) / (15 \text{ km/s})^2 = \underline{1.67 \times 10^{-17}}.$

(c) $\Delta E = (0.5)(6.0 \times 10^{24} \text{ kg})(2.5 \times 10^{-13} \text{ m/s})^2 = \underline{0.188 \text{ J}}.$

59. Conservation of momentum: $mv + MV = 0$

Conservation of energy: $mv^2/2 = 2(MV^2/2)$

$(m/M)v^2 = 2V^2 = 2(m/M)^2v^2, m/M = \underline{1/2}.$

60. This problem is unphysical, and has no solution.

61. The speed of the 1.5 kg mass at the bottom of the slope is $(2gh)^{\frac{1}{2}} = [(2)(9.8)(3.6)]^{\frac{1}{2}} = 8.4 \text{ m/s}.$
 Assuming a smooth curve to turn the mass without loss of energy, conservation of horizontal momentum gives $(1.5)(8.4 \text{ m/s}) = (1.5)v + 6V.$
 Conservation of energy gives $(1.5)(8.4)^2 = (1.5)v^2 + 6V^2.$ Solving gives

(a) $V = \underline{3.36 \text{ m/s}}. \quad v = \underline{-5.04 \text{ m/s}}.$

(b) We may now use conservation of energy $mv^2/2 = mgh$ to obtain $h = (5.04)^2/2/9.8 =$
 Distance up incline is $1.3/\sin 30^\circ = \underline{2.6 \text{ m}}.$

62. When m rebounds, it has speed v_0 (initially) back up slope. When it reaches bottom, v must be greater than V (the speed given M in the collision) to collide with M again. So, we need $v \geq V$ to meet the stated condition.

Set $v = -V$ as threshold.

$$mv_0 = -mv + MV = (M - m)v$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

$$mv_0^2 = mv^2 + Mv^2 = (m + M)v^2$$

Solving, $M = 3m$ at threshold. In general, need $M \geq 3m$